

Numerical Computing

Backpaper Examination

Instructions: All questions carry ten marks.

1. Define LU decomposition of a square matrix. Prove that for any square matrix A such a decomposition exists upto a permutation matrix. Give an example of a square matrix to prove that the permutation matrix is necessary for its LU decomposition.
2. Give three differences and two similarities between a script and a function in OCTAVE.
3. Write a script which asks for an integer, checks if it is so, and if an integer is given, it should report whether the given number is a prime number or not.
4. Write a function that asks for two arrays of length 10 and checks if any number occurs in both of them.
5. Palindrome is a number that remains same when read from front or back. For example, 121 is a palindrome. Write a function called PalindromeMaker that asks for a seven digit positive integer, and checks if any Palindrome can be generated using those seven digits or not.
6. Explain the Bisection Method of finding a root of continuous function. If this method is applied n times to obtain an approximate root, determine the maximum error of that solution with the actual root.
7. Explain the Newton-Raphson method of find a root of a function f . Give three different situations in which the initial choice will NOT converge to an actual root using this method.
8. If the points x_1, \dots, x_n are distinct, then prove that for any given y_1, \dots, y_n , there exists a unique polynomial P of degree at most $n - 1$ such that $P(x_i) = y_i$ for all $1 \leq i \leq n$.

9. Let f be a continuous function on the interval $[a, b]$. State and Prove the Basic Simpson's rule for numerically computing $\int_a^b f(x)dx$.
10. Define a *natural cubic spline* on an interval $[a, b]$. Derive the equations for the natural cubic spline that fits the following table:

x	-1	0	1
y	1	2	-1